Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: gp
To exit gp, type quit, \q, or <C-D> at prompt.

Help

describe function	?*function*
extended description	??*keyword*
list of relevant help topics	???*pattern*
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %``, etc.
n-th result since startup	%n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq_1; seq_2;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to *val*	default({d}, {*val*})
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n, \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r *filename*
set debuglevel for domain D to n	setdebug(D, n)

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({n}), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT. Integers; hex, binary	±31; ±0x1F, ±0b101
t_REAL. Reals	±3.14, 6.022 E23
t_INTMOD. Integers modulo m	Mod(n, m)
t_FRAC. Rational Numbers	n/m
t_FFELT. Elt in finite field $\mathbf{F}_q$	ffgen(q,'t)
t_COMPLEX. Complex Numbers	$x + y *$ I
t_PADIC. p-adic Numbers	x + O(p^k)
t_QUAD. Quadratic Numbers	$x + y *$ quadgen(D,{'w})
t_POLMOD. Polynomials modulo g	Mod(f, g)
t_POL. Polynomials	$a * x$^$n + \cdots + b$
t_SER. Power Series	f + O(x^k)
t_RFRAC. Rational Functions	f/g
t_QFB. Binary quadratic form	Qfb(a, b, c)
t_VEC/t_COL. Row/Column Vectors	[x, y, z], [x, y, z]~
t_VEC integer range	[1..10]
t_VECSMALL. Vector of small ints	Vecsmall([x, y, z])
t_MAT. Matrices	[a, b;c, d]
t_LIST. Lists	List([x, y, z])
t_STR. Strings	"abc"
t_INFINITY. $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	#x, sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p-adic, series prec. of x	padicprec(x, p), serprec(x, v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, - , *, /, ^, sqr
i←i+1, i←i-1, i←i*j, ...	i++, i--, i*=j,...
Euclidean quotient, remainder	x\/y, x\\y, x%y, divrem(x, y)
shift x left or right n bits	x<<n, x>>n or shift($x, \pm n$)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ===, lex, cmp
boolean operators (or, and, not)	||, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x, y), min(x, y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f, 2nd derivative, etc.	f', f'', ...
differential operator	diffop(f, v, d, {$n = 1$})
quote operator (formal variable)	'x
assignment	x = *value*
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n-th component of x	component(x, n)
n-th component of vector/list x	x[n]
components $a, a+1, \ldots, b$ of vector x	x[a..b]
(m, n)-th component of matrix x	x[m, n]
row m or column n of matrix x	x[m,], x[,n]
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec,Mat,Vecsmall
to list, set, map, string	List, Set, Map, Str
create (x mod y)	Mod(x, y)
make x a polynomial of v	Pol(x, {v})
variants of Pol *et al.*, in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser(x, {v})
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, strprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x, product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp(x, {p})
square of L^2 norm of x	norml2(x)
lift of x from Mods and p-adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (→t_INT)	liftint
lift all t_POLMOD (→t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)

intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
symmetric difference $x \Delta y$	setdelta(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch(x, y, {*flag*})
set of all $f(x, y)$, $x \in X$, $y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists. create empty list: L = List()

append x to list L	listput(L, x, {i})
remove i-th component from list L	listpop(L, {i})
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort(L, {*flag*})

Maps. create empty dictionary: M = Map()

attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict? (set v to $M(k)$)	mapisdefined(M, k, {&v})
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to Pi if parameter omitted;
z, t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); *seq*

fun = (x, y=Pi) -> my(z=1, t); *seq*

attach help message h to s	addhelp(s, h)
undefine symbol s (also kills help)	kill(s)

Control Statements (X: formal parameter in expression *seq*)

if $a \neq 0$, evaluate seq_1, else seq_2	if(a, {seq_1}, {seq_2})
eval. *seq* for $a \le X \le b$	for($X = a, b, seq$)
...for $X \in v$	foreach(v, X, seq)
...for primes $a \le X \le b$	forprime($X = a, b, seq$)
...for primes $\equiv a \pmod{q}$	forprimestep($X = a, b, q, seq$)
...for composites $a \le X \le b$	forcomposite($X = a, b, seq$)
...for $a \le X \le b$ stepping s	forstep($X = a, b, s, seq$)
...for X dividing n	fordiv(n, X, seq)
...$X = [n, factor(n)]$, $a \le n \le b$	forfactored($X = a, b, seq$)
...as above, n squarefree	forsquarefree($X = a, b, seq$)
...$X = [d, factor(d)]$, $d \mid n$	fordivfactored(n, X, seq)
multivariable for, lex ordering	forvec($X = v, seq$)

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loop over partitions of n	`forpart(p = n, seq)`
... permutations of S	`forperm(S, p, seq)`
... subsets of $\{1, \dots, n\}$	`forsubset(n, p, seq)`
... k-subsets of $\{1, \dots, n\}$	`forsubset([n, k], p, seq)`
... vectors v, $q(v) \le B$; $q > 0$	`forqfvec(v, q, b, seq)`
... $H < G$ finite abelian group	`forsubgroup(H = G)`
evaluate *seq* until $a \ne 0$	`until(a, seq)`
while $a \ne 0$, evaluate *seq*	`while(a, seq)`
exit n innermost enclosing loops	`break({n})`
start new iteration of n-th enclosing loop	`next({n})`
return x from current subroutine	`return({x})`

Exceptions, warnings

raise an exception / warning	`error(), warning()`
type of error message E	`errname(E)`
try seq_1, evaluate seq_2 on error	`iferr(seq1, E, seq2)`

Functions with closure arguments / results

number of arguments of f	`arity(f)`
select from v according to f	`select(f, v)`
apply f to all entries in v	`apply(f, v)`
evaluate $f(a_1, \dots, a_n)$	`call(f, a)`
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$	`fold(f, a)`
calling function as closure	`self()`

Sums & Products

sum $X = a$ to $X = b$, initialized at x	`sum(X = a, b, expr, {x})`
sum entries of vector v	`vecsum(v)`
product of all vector entries	`vecprod(v)`
sum *expr* over divisors of n	`sumdiv(n, X, expr)`
... assuming *expr* multiplicative	`sumdivmult(n, X, expr)`
product $a \le X \le b$, initialized at x	`prod(X = a, b, expr, {x})`
product over primes $a \le X \le b$	`prodeuler(X = a, b, expr)`

Sorting

sort x by k-th component	`vecsort(x, {k}, {fl = 0})`
min. m of x ($m = x[i]$), max.	`vecmin(x, {&i}), vecmax`
does y belong to x, sorted wrt. f	`vecsearch(x, y, {f})`
$\prod g^x \to$ factorization ($\Rightarrow$ sorted, unique g)	`matreduce(m)`

Input/Output

print with/without `\n`, TeX format	`print, print1, printtex`
pretty print matrix	`printp`
print fields with separator	`printsep(sep,...),, printsep1`
formatted printing	`printf()`
write *args* to file	`write, write1, writetex(file, args)`
write x in binary format	`writebin(file, x)`
read file into GP	`read({file})`
... return as vector of lines	`readvec({file})`
... return as vector of strings	`readstr({file})`
read a string from keyboard	`input()`

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in `r`(ead), `w`(rite) or `a`(ppend) mode.

get descriptor n for file *path* in given *mode*	`fileopen(path, mode)`
... from shell *cmd* output (pipe)	`fileextern(cmd)`
close descriptor	`fileclose(n)`
commit pending write operations	`fileflush(n)`
read logical line from file	`fileread(n)`
... raw line from file	`filereadstr(n)`
write `s\n` to file	`filewrite(n, s)`
... write s to file	`filewrite1(n, s)`

Timers

CPU time in *ms* and reset timer	`gettime()`
CPU time in *ms* since gp startup	`getabstime()`
time in *ms* since UNIX Epoch	`getwalltime()`
timeout command after s seconds	`alarm(s, expr)`

Interface with system

allocates a new stack of s bytes	`allocatemem({s})`
alias *old* to *new*	`alias(new, old)`
install function from library	`install(f, code, {gpf}, {lib})`
execute system command a	`system(a)`
... and feed result to GP	`extern(a)`
... returning GP string	`externstr(a)`
get `$VAR` from environment	`getenv("VAR")`
expand env. variable in string	`strexpand(x)`

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use `export` for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$	`parapply(f, x)`
evaluate closures $f[1], \dots, f[n]$	`pareval(f)`
as `select`	`parselect(f, A, {flag})`
as `sum`	`parsum(i = a, b, expr)`
as `vector`	`parvector(n, i, {expr})`
eval f for $i = a, \dots, b$	`parfor(i = a, {b}, f, {r}, {f2})`
... for each element x in v	`parforeach(v, x, f, {r}, {f2})`
... for p prime in $[a, b]$	`parforprime(p = a, {b}, f, {r}, {f2})`
... for $p = a \bmod q$	`parforprimestep(p = a, {b}, q, f, {r}, {f2})`
... multivariate	`parforvec(X = v, f, {r}, {f2}, {flag})`
export x to parallel world	`export(x)`
... all dynamic variables	`exportall()`
frees exported value x	`unexport(x)`
... all exported values	`unexportall()`

Linear Algebra

dimensions of matrix x	`matsize(x)`
multiply two matrices	`x * y`
... assuming result is diagonal	`matmultodiagonal(x, y)`
concatenation of x and y	`concat(x, {y})`
extract components of x	`vecextract(x, y, {z})`
transpose of vector or matrix x	`x~, mattranspose(x)`
adjoint of the matrix x	`matadjoint(x)`
eigenvectors/values of matrix x	`mateigen(x)`
characteristic/minimal polynomial of x	`charpoly(x), minpoly(x)`
trace/determinant of matrix x	`trace(x), matdet(x)`
permanent of matrix x	`matpermanent(x)`
Frobenius form of x	`matfrobenius(x)`
QR decomposition	`matqr(x)`
apply `matqr`'s transform to v	`mathouseholder(Q, v)`

Constructors & Special Matrices

<table>
<tr><td>$\{g(x) \colon x \in v \text{ s.t. } f(x)\}$</td><td><code>[g(x) | x <- v, f(x)]</code></td></tr>
<tr><td>$\{x \colon x \in v \text{ s.t. } f(x)\}$</td><td><code>[x | x <- v, f(x)]</code></td></tr>
<tr><td>$\{g(x) \colon x \in v\}$</td><td><code>[g(x) | x <- v]</code></td></tr>
<tr><td>row vec. of expr eval'ed at $1 \le i \le n$</td><td><code>vector(n, {i}, {expr})</code></td></tr>
<tr><td>col. vec. of expr eval'ed at $1 \le i \le n$</td><td><code>vectorv(n, {i}, {expr})</code></td></tr>
<tr><td>vector of small ints</td><td><code>vectorsmall(n, {i}, {expr})</code></td></tr>
<tr><td>$[c, c \cdot x, \dots, c \cdot x^n]$</td><td><code>powers(x, n, {c = 1})</code></td></tr>
<tr><td>$[1, 2^x, \dots, n^x]$</td><td><code>dirpowers(n, x)</code></td></tr>
<tr><td>matrix $1 \le i \le m$, $1 \le j \le n$</td><td><code>matrix(m, n, {i}, {j}, {expr})</code></td></tr>
<tr><td>define matrix by blocks</td><td><code>matconcat(B)</code></td></tr>
<tr><td>diagonal matrix with diagonal x</td><td><code>matdiagonal(x)</code></td></tr>
<tr><td>is x diagonal?</td><td><code>matisdiagonal(x)</code></td></tr>
<tr><td>$x \cdot$ <code>matdiagonal</code>(d)</td><td><code>matmuldiagonal(x, d)</code></td></tr>
<tr><td>$n \times n$ identity matrix</td><td><code>matid(n)</code></td></tr>
<tr><td>Hessenberg form of square matrix x</td><td><code>mathess(x)</code></td></tr>
<tr><td>$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$</td><td><code>mathilbert(n)</code></td></tr>
<tr><td>$n \times n$ Pascal triangle</td><td><code>matpascal(n - 1)</code></td></tr>
<tr><td>companion matrix to polynomial x</td><td><code>matcompanion(x)</code></td></tr>
<tr><td>Sylvester matrix of x and y</td><td><code>polsylvestermatrix(x, y)</code></td></tr>
</table>

Gaussian elimination

kernel of matrix x	`matker(x, {flag})`
intersection of column spaces of x and y	`matintersect(x, y)`
solve $MX = B$ (M invertible)	`matsolve(M, B)`
one sol of $M * X = B$	`matinverseimage(M, B)`
basis for image of matrix x	`matimage(x)`
columns of x *not* in `matimage`	`matimagecompl(x)`
supplement columns of x to get basis	`matsupplement(x)`
rows, cols to extract invertible matrix	`matindexrank(x)`
rank of the matrix x	`matrank(x)`
solve $MX = B \bmod D$	`matsolvemod(M, D, B)`
image mod D	`matimagemod(M, D)`
kernel mod D	`matkermod(M, D)`
inverse mod D	`matinvmod(M, D)`
determinant mod D	`matdetmod(M, D)`

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t xQy$	`qfeval({Q = id}, x, y)`
evaluate ${}^t xQx$	`qfeval({Q = id}, x)`
signature of quad form ${}^t y * x * y$	`qfsign(x)`
decomp into squares of ${}^t y * x * y$	`qfgaussred(x)`
eigenvalues/vectors for real symmetric x	`qfjacobi(x)`

HNF and SNF

upper triangular Hermite Normal Form	`mathnf(x)`
HNF of x where d is a multiple of $\det(x)$	`mathnfmod(x, d)`
multiple of $\det(x)$	`matdetint(x)`
HNF of (x | `diagonal`(D))	`mathnfmodid(x, D)`
elementary divisors of x	`matsnf(x)`
q-rank from elementary divisors	`snfrank(v, q)`
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	`poldiscreduced(f)`
integer kernel of x	`matkerint(x)`
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz(x, p)`

Lattices

LLL-algorithm applied to columns of x	`qflll(x, {flag})`
... for Gram matrix of lattice	`qflllgram(x, {flag})`
find up to m sols of `qfnorm`$(x, y) \le b$	`qfminim(x, b, m)`
v, $v[i]$:=number of y s.t. `qfnorm`$(x, y) = i$	`qfrep(x, B, {flag})`
perfection rank of x	`qfperfection(x)`
find isomorphism between q and Q	`qfisom(q, Q)`
precompute for isomorphism test with q	`qfisominit(q)`
automorphism group of q	`qfauto(q)`

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩

convert `qfauto` for GAP/Magma	`qfautoexport(G,{flag})`
orbits of V under $G \subset \mathrm{GL}(V)$	`qforbits(G,V)`

Polynomials & Rational Functions

all defined polynomial variables	`variables()`
get var. of highest priority (higher than v)	`varhigher(name,{v})`
...of lowest priority (lower than v)	`varlower(name,{v})`

Coefficients, variables and basic operators

degree of f	`poldegree(f)`
coef. of degree n of f, leading coef.	`polcoef(f,n)`, `pollead`
main variable / all variables in f	`variable(f)`, `variables(f)`
replace x by y in f	`subst(f,x,y)`
evaluate f replacing vars by their value	`eval(f)`
replace polynomial expr. $T(x)$ by y in f	`substpol(f,T,y)`
replace $x_1,\dots,x_n$ by $y_1,\dots,y_n$ in f	`substvec(f,x,y)`
$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$	`polrecip(f)`
gcd of coefficients of f	`content(f)`
derivative of f w.r.t. x	`deriv(f,{x})`
...n-th derivative of f	`derivn(f,n,{x})`
formal integral of f w.r.t. x	`intformal(f,{x})`
formal sum of f w.r.t. x	`sumformal(f,{x})`

Constructors & Special Polynomials

interpolation polynomial at $(x[1],y[1]),\dots,(x[n],y[n])$, evaluated at t, with error estimate e	`polinterpolate(x,{y},{t},{&e})`
T_n/U_n, H_n	`polchebyshev(n)`, `polhermite(n)`
P_n, $L_n^{(\alpha)}$	`pollegendre(n)`, `pollaguerre(n,a)`
n-th cyclotomic polynomial Φ_n	`polcyclo(n)`
return n if $f = \Phi_n$, else 0	`poliscyclo(f)`
is f a product of cyclotomic polynomials?	`poliscycloprod(f)`
Zagier's polynomial of index (n,m)	`polzagier(n,m)`

Resultant, elimination

discriminant of polynomial f	`poldisc(f)`
find factors of `poldisc(f)`	`poldiscfactors(f)`
resultant $R = \mathrm{Res}_v(f,g)$	`polresultant(f,g,{v})`
$[u,v,R]$, $xu + yv = \mathrm{Res}_v(f,g)$	`polresultantext(x,y,{v})`
solve Thue equation $f(x,y) = a$	`thue(t,a,{sol})`
initialize t for Thue equation solver	`thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of f	`polroots(f)`
bound complex roots of f	`polrootsbound(f)`
number of real roots of f (in $[a,b]$)	`polsturm(f,{[a,b]})`
real roots of f (in $[a,b]$)	`polrootsreal(f,{[a,b]})`
complex embeddings of `t_POLMOD` z	`conjvec(z)`

Roots and Factorization (Finite fields)

factor f mod p, roots	`factormod(f,p)`, `polrootsmod`
factor f over $\mathbf{F}_p[x]/(T)$, roots	`factormod(f,[T,p])`, `polrootsmod`
squarefree factorization of f in $\mathbf{F}_q[x]$	`factormodSQF(f,{D})`
distinct degree factorization of f in $\mathbf{F}_q[x]$	`factormodDDF(f,{D})`
factor n-th cyclotomic pol. Φ_n mod p	`factormodcyclo(n,p)`

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots	`factorpadic(f,p,r)`, `polrootspadic`
p-adic root of f congruent to a mod p	`padicappr(f,a)`
Newton polygon of f for prime p	`newtonpoly(f,p)`
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i]$ mod p^e	`polhensellift(A,B,p,e)`
$T = \prod(x - z_i) \mapsto \prod(x - \omega(z_i)) \in \mathbf{Z}_p[x]$	`polteichmuller(T,p,e)`
extensions of $\mathbf{Q}_p$ of degree N	`padicfields(p,N)`

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	`polsym(f,n)`
Graeffe transform of f, $g(x^2) = f(x)f(-x)$	`polgraeffe(f)`
factor f over coefficient field	`factor(f)`
cyclotomic factors of $f \in \mathbf{Q}[X]$	`polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).

find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	`ffinit(p,n,{x})`
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	`t = ffgen(T,'t)`
...indirectly, with implicit T	`t = ffgen(q,'t); T = t.mod`
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	`m = ffembed(a,b)`
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$,	`ffextend(a,P)`
evaluate map m on x	`ffmap(m,x)`
inverse map of m	`ffinvmap(m)`
compose maps $m \circ n$	`ffcompomap(m,n)`
x as polmod over codomain of map m	`ffmaprel(m,x)`
F^n over $\mathbf{F}_q \ni a$	`fffrobenius(a,n)`
$\#\{$monic irred. $T \in \mathbf{F}_q[x], \deg T = n\}$	`ffnbirred(q,n)`

Formal & p-adic Series

truncate power series or p-adic number	`truncate(x)`
valuation of x at p	`valuation(x,p)`

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	`taylor(f,x)`
Laurent series of closure F up to x^k	`laurentseries(f,k)`
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	`serconvol(a,b)`
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	`serlaplace(f)`
reverse power series F so $F(f(x)) = x$	`serreverse(f)`
remove terms of degree $< n$ in f	`serchop(f,n)`
Dirichlet series multiplication / division	`dirmul`, `dirdiv(x,y)`
Dirichlet Euler product (b terms)	`direuler(p=a,b,expr)`

Transcendental and p-adic Functions

real, imaginary part of x	`real(x)`, `imag(x)`
absolute value, argument of x	`abs(x)`, `arg(x)`
square/nth root of x	`sqrt(x)`, `sqrtn(x,n,{&z})`
all n-th roots of 1	`rootsof1(n)`
FFT of $[f_0,\dots,f_{n-1}]$	w `= fftinit(n)`, `fft/fftinv(w,f)`
trig functions	`sin, cos, tan, cotan, sinc`
inverse trig functions	`asin, acos, atan`
hyperbolic functions	`sinh, cosh, tanh, cotanh`
inverse hyperbolic functions	`asinh, acosh, atanh`
$\log(x)$, $\log(1+x)$, e^x, $e^x - 1$	`log, log1p, exp, expm1`
Euler Γ function, $\log\Gamma$, Γ'/Γ	`gamma, lngamma, psi`
half-integer gamma function $\Gamma(n+1/2)$	`gammah(n)`
Riemann's zeta $\zeta(s) = \sum n^{-s}$	`zeta(s)`
$\sum_{1 \le n \le N} n^s$	`dirpowerssum(N,s)`
Hurwitz's $\zeta(s,x) = \sum (n+x)^{-s}$	`zetahurwitz(s,x)`
Lerch $\Phi(z,s,x) = \sum z^n (n+x)^{-s}$	`lerchphi(z,s,x)`
Lerch $L(s,x,t) = \Phi(e^{2i\pi t},s,x)$	`lerchzeta(s,x,t)`
multiple zeta value (MZV), $\zeta(s_1,\dots,s_k)$	`zetamult(s,{T})`
all MZVs for weight $\sum s_i = n$	`zetamultall(n)`
convert MZV id to $[s_1,\dots,s_k]$	`zetamultconvert(f,{flag})`
MZV dual sequence	`zetamultdual(s)`
multiple polylog $Li_{s_1,\dots s_k}(z_1,\dots,z_k)$	`polylogmult(s,z)`

incomplete Γ function ($y = \Gamma(s)$)	`incgam(s,x,{y})`
complementary incomplete Γ	`incgamc(s,x)`
$\int_x^\infty e^{-t}\,dt/t$, $(2/\sqrt{\pi})\int_x^\infty e^{-t^2}dt$	`eint1, erfc`
elliptic integral of 1st and 2nd kind	`ellK(k)`, `ellE(k)`
dilogarithm of x	`dilog(x)`
m-th polylogarithm of x	`polylog(m,x,{flag})`
U-confluent hypergeometric function	`hyperu(a,b,u)`
Hypergeometric ${}_pF_q(A,B;z)$	`hypergeom(A,B,z)`
Bessel $J_n(x)$, $J_{n+1/2}(x)$	`besselj(n,x)`, `besseljh(n,x)`
Bessel I_ν, K_ν, H_ν^1, H_ν^2, Y_ν	`(bessel)i, k, h1, h2, y`
k-th zero of $J_\nu(x)$	`besseljzero(nu,{k=1})`
k-th zero of $Y_\nu(x)$	`besselyzero(nu,{k=1})`
Airy functions $A_i(x)$, $B_i(x)$	`airy(x)`
Lambert W: x s.t. $xe^x = y$	`lambertw(y)`
Teichmuller character of p-adic x	`teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm$`oo` (decreasing at least as x^{-2}) or

$(x-a)^{-\alpha}$ singularity	$[a,\alpha]$
exponential decrease $e^{-\alpha\lvert x\rvert}$	$[\pm\infty,\alpha]$, $\alpha > 0$
slow decrease $\lvert x\rvert^\alpha$	$\dots\alpha < -1$
oscillating as $\cos(kx)$)	$\alpha = k$`I`, $k > 0$
oscillating as $\sin(kx)$)	$\alpha = -k$`I`, $k > 0$
numerical integration	`intnum(x=a,b,f,{T})`
weights T for `intnum`	`intnuminit(a,b,{m})`
weights T incl. kernel K	`intfuncinit(t=a,b,K,{m})`
integrate $(2i\pi)^{-1}f$ on circle $\lvert z-a\rvert = R$	`intcirc(x=a,R,f,{T})`

Other integration methods

n-point Gauss-Legendre	`intnumgauss(x=a,b,f,{n})`
weights for n-point Gauss-Legendre	`intnumgaussinit({n})`
quasi-periodic function, period $2H$	`intnumosc(x=a,f,H)`
Romberg (low accuracy)	`intnumromb(x=a,b,f,{flag})`

Numerical summation

sum of series $f(n)$, $n \ge a$ (low accuracy)	`suminf(n=a,expr)`
sum of alternating/positive series	`sumalt, sumpos`
sum of series using Euler-Maclaurin	`sumnum(n=a,f,{T})`
...Sidi summation	`sumnumsidi(n=a,f)`
$\sum_{n \ge a} F(n)$, F rational function	`sumnumrat(F,a)`
...$\sum_{p \ge a} F(p^s)$	`sumeulerrat(F,{s=1},{a=2})`
weights for `sumnum`, a as in DE	`sumnuminit({∞,a})`
sum of series by Monien summation	`sumnummonien(n=a,f,{T})`
weights for `sumnummonien`	`sumnummonieninit({∞,a})`
sum of series using Abel-Plana	`sumnumap(n=a,f,{T})`
weights for `sumnumap`, a as in DE	`sumnumapinit({∞,a})`
sum of series using Lagrange	`sumnumlagrange(n=a,f,{T})`
weights for `sumnumlagrange`	`sumnumlagrangeinit`

Products

product $a \le X \le b$, initialized at x	`prod(X=a,b,expr,{x})`
product over primes $a \le X \le b$	`prodeuler(X=a,b,expr)`
infinite product $a \le X \le \infty$	`prodinf(X=a,expr)`
$\prod_{n \ge a} F(n)$, F rational function	`prodnumrat(F,a)`
$\prod_{p \ge a} F(p^s)$	`prodeulerrat(F,{s=1},{a=2})`

Other numerical methods

real root of f in $[a,b]$; bracketed root	`solve(X = a, b, f)`
... interval splitting, step s	`solvestep(X = a, b, s, f, {flag = 0})`
limit of $f(t)$, $t \to \infty$	`limitnum(f, {alpha})`
asymptotic expansion of f (rational)	`asympnum(f, {alpha})`
... $N+1$ terms as floats	`asympnumraw(f, N, {alpha})`
numerical derivation w.r.t x: $f'(a)$	`derivnum(x = a, f)`
evaluate continued fraction F at t	`contfraceval(F, t, {L})`
power series to cont. fraction (L terms)	`contfracinit(S, {L})`
Padé approximant (deg. denom. $\leq B$)	`bestapprPade(S, {B})`

Elementary Arithmetic Functions

vector of binary digits of $\lvert x \rvert$	`binary(x)`
bit number n of integer x	`bittest(x, n)`
Hamming weight of integer x	`hammingweight(x)`
digits of integer x in base B	`digits(x, {B = 10})`
sum of digits of integer x in base B	`sumdigits(x, {B = 10})`
integer from digits	`fromdigits(v, {B = 10})`
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round(x, {&e})`
truncate x	`truncate(x, {&e})`
gcd/LCM of x and y	`gcd(x, y)`, `lcm(x, y)`
gcd of entries of a vector/matrix	`content(x)`

Primes and Factorization

extra prime table	`addprimes()`
add primes in v to prime table	`addprimes(v)`
remove primes from prime table	`removeprimes(v)`
Chebyshev $\pi(x)$, n-th prime p_n	`primepi(x)`, `prime(n)`
vector of first n primes	`primes(n)`
smallest prime $\geq x$	`nextprime(x)`
largest prime $\leq x$	`precprime(x)`
factorization of x	`factor(x, {lim})`
... selecting specific algorithms	`factorint(x, {flag = 0})`
$n = df^2$, d squarefree/fundamental	`core(n, {fl})`, `coredisc`
certificate for (prime) N	`primecert(N)`
verifies a certificate c	`primecertisvalid(c)`
convert certificate to Magma/PRIMO	`primecertexport`
recover x from its factorization	`factorback(f, {e})`
$x \in \mathbf{Z}$, $\lvert x \rvert \leq X$, $\gcd(N, P(x)) \geq N$	`zncoppersmith(P, N, X, {B})`
divisors of N in residue class r mod s	`divisorslenstra(N, r, s)`

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega(n)`, `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors(n)`, `numdiv`
sum of (k-th powers of) divisors of n	`sigma(n, {k})`
Möbius μ-function	`moebius(x)`
Ramanujan's τ-function	`ramanujantau(x)`

Combinatorics

factorial of x	`x!` or `factorial(x)`
binomial coefficient $\binom{x}{k}$	`binomial(x, {k})`
Bernoulli number B_n as real/rational	`bernreal(n)`, `bernfrac`
$[B_0, B_2, \ldots B_{2k}]$	`bernvec(k)`
Bernoulli polynomial $B_n(x)$	`bernpol(n, {x})`
Euler numbers	`eulerfrac`, `eulerreal`, `eulervec`
Euler polynomial $E_n(x)$	`eulerpol(n, {x})`
Eulerian polynomial $A_n(x)$	`eulerianpol`
Fibonacci number F_n	`fibonacci(n)`
Harmonic number $H_{n,r} = 1^{-r} + \ldots + n^{-r}$	`harmonic(n, r)`
Stirling numbers $s(n,k)$ and $S(n,k)$	`stirling(n, k, {flag})`

Pari-GP reference card

(PARI-GP version 2.15.3)

number of partitions of n	`numbpart(n)`
k-th permutation on n letters	`numtoperm(n, k)`
... index k of permutation v	`permtonum(v)`
order of permutation p	`permorder(p)`
signature of permutation p	`permsign(p)`
cyclic decomposition of permutation p	`permcycles(p)`

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi(x)`
multiplicative order of x (divides o)	`znorder(x, {o})`, `fforder`
primitive root mod q / `x.mod`	`znprimroot(q)`, `ffprimroot(x)`
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar(n)`
discrete logarithm of x in base g	`znlog(x, g, {o})`, `fflog`
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker(x, y)`
quadratic Hilbert symbol (at p)	`hilbert(x, y, {p})`

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	`chinese(x, y)`
minimal u, v so $xu + yv = \gcd(x, y)$	`gcdext(x, y)`
half-gcd algorithm	`halfgcd(x, y)`
continued fraction of x	`contfrac(x, {b}, {lmax})`
last convergent of continued fraction x	`contfracpnqn(x)`
rational approximation to x (den. $\leq B$)	`bestappr(x, {B})`
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	`bestapprnf(x, T)`

Miscellaneous

integer square / n-th root of x	`sqrtint(x)`, `sqrtnint(x, n)`
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	`logint(x, b, {&z})`

Characters

Let $cyc = [d_1, \ldots, d_k]$ represent an abelian group $G = \oplus(\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar(q, 1)` for Dirichlet characters. A character χ is coded by $[c_1, \ldots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.

$\chi \cdot \psi$; χ^{-1}; $\chi \cdot \psi^{-1}$; χ^k	`charmul`, `charconj`, `chardiv`, `charpow`
order of χ	`charorder(cyc, χ)`
kernel of χ	`charker(cyc, χ)`
$\chi(x)$, G a GP group structure	`chareval(G, χ, x, {z})`
Galois orbits of characters	`chargalois(G)`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	`G = znstar(q, 1)`
convert datum D to $[G, \chi]$	`znchar(D)`
is χ odd?	`zncharisodd(G, χ)`
real $\chi \to$ Kronecker symbol $(D/.)$	`znchartokronecker(G, χ)`
conductor of χ	`zncharconductor(G, χ)`
$[G_0, \chi_0]$ primitive attached to χ	`znchartoprimitive(G, χ)`
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	`zncharinduce(G, χ, N)`
χ_p	`znchardecompose(G, χ, p)`
$\prod_{p \mid (Q,N)} \chi_p$	`znchardecompose(G, χ, Q)`
complex Gauss sum $G_a(\chi)$	`znchargauss(G, χ)`

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \to$ character	`znconreychar(G, m)`
character $\to$ Conrey label	`znconreyexp(G, χ)`
log on Conrey generators	`znconreylog(G, m)`
conductor of χ (χ_0 primitive)	`znconreyconductor(G, χ, {χ0})`

True-False Tests

is x the disc. of a quadratic field?	`isfundamental(x)`
is x a prime?	`isprime(x)`
is x a strong pseudo-prime?	`ispseudoprime(x)`
is x square-free?	`issquarefree(x)`
is x a square?	`issquare(x, {&n})`
is x a perfect power?	`ispower(x, {k}, {&n})`
is x a perfect power of a prime? ($x = p^n$)	`isprimepower(x, &n)`
... of a pseudoprime?	`ispseudoprimepower(x, &n)`
is x powerful?	`ispowerful(x)`
is x a totient? ($x = \varphi(n)$)	`istotient(x, {&n})`
is x a polygonal number? ($x = P(s, n)$)	`ispolygonal(x, s, {&n})`
is *pol* irreducible?	`polisirreducible(pol)`

Graphic Functions

crude graph of *expr* between a and b	`plot(X = a, b, expr)`

High-resolution plot (immediate plot)

plot *expr* between a and b	`ploth(X = a, b, expr, {flag}, {n})`
plot points given by lists lx, ly	`plothraw(lx, ly, {flag})`
terminal dimensions	`plothsizes()`

Rectwindow functions

init window w, with size x,y	`plotinit(w, x, y)`
erase window w	`plotkill(w)`
copy w to w_2 with offset (dx, dy)	`plotcopy(w, w2, dx, dy)`
clips contents of w	`plotclip(w)`
scale coordinates in w	`plotscale(w, x1, x2, y1, y2)`
`ploth` in w	`plotrecth(w, X = a, b, expr, {flag}, {n})`
`plothraw` in w	`plotrecthraw(w, data, {flag})`
draw window w_1 at (x_1, y_1), ...	`plotdraw([[w1, x1, y1], ...])`

Low-level Rectwindow Functions

set current drawing color in w to c	`plotcolor(w, c)`
current position of cursor in w	`plotcursor(w)`
write s at cursor's position	`plotstring(w, s)`
move cursor to (x, y)	`plotmove(w, x, y)`
move cursor to $(x + dx, y + dy)$	`plotrmove(w, dx, dy)`
draw a box to (x_2, y_2)	`plotbox(w, x2, y2)`
draw a box to $(x + dx, y + dy)$	`plotrbox(w, dx, dy)`
draw polygon	`plotlines(w, lx, ly, {flag})`
draw points	`plotpoints(w, lx, ly)`
draw line to $(x + dx, y + dy)$	`plotrline(w, dx, dy)`
draw point $(x + dx, y + dy)$	`plotrpoint(w, dx, dy)`

Convert to Postscript or Scalable Vector Graphics

The format f is either `"ps"` or `"svg"`.

as `ploth`	`plothexport(f, X = a, b, expr, {flag}, {n})`
as `plothraw`	`plothrawexport(f, lx, ly, {flag})`
as `plotdraw`	`plotexport(f, [[w1, x1, y1], ...])`

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